NUMERICAL STUDY OF THE FILLING OF A CAVITY FROM AN EXPLOSIVE PLASMA GENERATOR

A. E. Voĭtenko and É. A. Zel'dina

UDC 533.6+535.231.6

An explosive plasma generator (EPG) [1] is often employed to fill a cavity with a dense plasma, for example, in the study of intense ablation regimes, in explosive lamps, in explosive plasmachemical sysnthesis, etc. Here the apparatus consists of an EPG connected by a pipe (plasma conduit) with working cavity [2-5].

The difficulty of performing measurements in a nonstationary flow of dense plasma leads to the fact that experimental information on the operation of these devices is incomplete. Usually the mass and energy of the plasma are determined experimentally, and sometimes the pressure, temperature, velocity, and heat fluxes are measured at separate points [1-5]. A more complete description of the processes occurring can be obtained by combined use of experimental data and numerical calculations.

Numerical simulation of the EPG itself was performed in [6-8], and the radiative characteristics of the explosive lamp are studied in [9]. Taking into account radiant heat transfer [8, 9] gives a more complete description of the flow of dense plasma, but it is laborious and expensive with the use of computers.

In a number of practical problems Boltzmann's number Bo >> 1, which makes it possible to neglect radiant heat transfer and thereby simplify the mathematical model of the process.

In this work the motion of the plasma in the pipe and the filling of the cavity with the plasma are studied by the numerical method of S. K. Godunov.

We are interested in a nonstationary, two-dimensional, axisymmetric plasma flow in a pipe and cylindrial cavity. The flow is assumed to be adiabatic and the plasma is assumed to be nonviscous and thermally nonconducting. The equilibrium ionization is taken into account by introducing into the equation of state a constant effective adiabatic index.

The system of Euler's gas-dynamic equations in cylindrical coordinates has the form

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial r}=-\frac{\rho v}{r},\ \frac{\partial(\rho u)}{\partial t}+\frac{\partial(p+\rho u^2)}{\partial x}+\frac{\partial(\rho uv)}{\partial r}=-\frac{\rho uv}{r},$$
$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(p+\rho v^2)}{\partial r}=-\frac{\rho v^2}{r},\ \frac{\partial(\rho E)}{\partial t}+\frac{\partial(\rho uH)}{\partial x}+\frac{\partial(\rho vH)}{\partial r}= \tag{1}$$
$$=-\frac{\rho vH}{r},\ E=\varepsilon+\frac{u^2+v^2}{2},\ H=E+\frac{p}{\rho},\ \varepsilon=\frac{p}{(\gamma-1)\rho}.$$

Here ρ and p are the density and pressure of the plasma; ϵ is the specific internal energy; u and v are the components of the velocity along the x and r coordiante axes; t is the time; x and r are the longitudinal and radial coordinates; and, γ is the effective adiabatic index.

The system (1) is supplemented by Raĭzer's equations [10] for determining the temperature and degree of ionization of the plasma

$$I\left(m+\frac{1}{2}\right)=kT\ln\frac{AT^{3/2}}{mN\rho},\ \varepsilon=\frac{3}{2}N(1+m)kT+NQ(m), \tag{2}$$
$$p=N\rho(1+m)kT,$$

where m is the degree of ionization; I_m is the ionization potential; T is the temperature; N is the number of ions per unit mass; $Q_m=\Sigma I_m$ is the energy of detachment of m electrons from an atom; k is the Boltzmann constant; $A=4.8\cdot10^{21}$ deg$^{-3/2}\cdot$m^{-3} is a constant.

Dnepropetrovsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 16-20, January-February, 1988. Original article submitted October 15, 1986.

 0021-8944/88/2901-0014$12.50

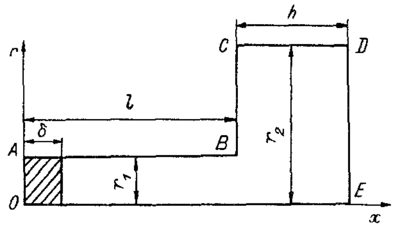

Fig. 1

The equations (1) are integrated in a region (Fig. 1) consisting of the pipe OAB and the cavity BCDE. The boundary conditions on the walls of the pipe, on the walls of the cavity, and on the symmetry axis OE require that the normal velocity component vanish.

It is assumed that initially the plasmoid occupies some volume ($0 \le x \le \delta$) in the pipe. The initial conditions for the plasma are: $\varepsilon(0, x, r) = \varepsilon_0$, $\rho(0, x, r) = \rho_0$, $u(0, x, r) = v(0, x, r) = 0$. The pipe outside the plasmoid and the cavity are filled at t = 0 with a cold stationary gas or they are evacuated.

The starting behavior of the plasma p_0 and the effective adiabatic index γ, which is assumed to be constant in what follows, are found from the given initial conditions ϵ_0 and ρ_0 from the solution of the system (2).

The system of gas-dynamic equations (1) can be solved numerically using Godunov's finite-difference schemes [11] on a stationary rectangular grid, oriented along the coordinate axes. This scheme has the feature that there is a decoupling along the coordinate axes, enabling integration of the system (1) first along the direction x over a half time step and then over r over the second half step.

The finite difference equations for the system (1) have the following form along the Ox axis

$$\overset{n+\frac{1}{2}}{\sigma}_{i-\frac{1}{2},j-\frac{1}{2}} = \overset{n}{\sigma}_{i-\frac{1}{2},j-\frac{1}{2}} - \left(A_{i,j-\frac{1}{2}} - A_{i-1,j-\frac{1}{2}}\right)\frac{\Delta t}{\Delta x},$$

and along the Or axis

$$\sigma^{n+1}_{i-\frac{1}{2},j-\frac{1}{2}} = \overset{n+\frac{1}{2}}{\sigma}_{i-\frac{1}{2},j-\frac{1}{2}} - \left(B_{i-\frac{1}{2},j} - B_{i-\frac{1}{2},j-1}\right)\frac{\Delta t}{\Delta r} - \Psi_{i-\frac{1}{2},j-\frac{1}{2}}\Delta t,$$

$$\sigma = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{bmatrix}, \quad A = \begin{bmatrix} RU \\ P + RU^2 \\ RUV \\ PU + RU\left(\frac{P}{(\gamma-1)R} + \frac{U^2+V^2}{2}\right) \end{bmatrix},$$

$$B = \begin{bmatrix} RV \\ RUV \\ P + RV^2 \\ PV + RV\left(\frac{P}{(\gamma-1)R} + \frac{U^2+V^2}{2}\right) \end{bmatrix},$$

$$\Psi = \begin{bmatrix} 0.5\,[(RV)_1 + (RV)_2] \\ 0.5\,[(RUV)_1 + (RUV)_2] \\ 0.5\,[(P+RV^2)_1 + (P+RV^2)_2] \\ 0.5\left[\left(\left(\frac{P}{(\gamma-1)R} + \frac{U^2+V^2}{2} + \frac{P}{R}\right)RV\right)_1 + \left(\left(\frac{P}{(\gamma-1)R} + \frac{U^2+V^2}{2} + \frac{P}{R}\right)RV\right)_2\right] \end{bmatrix}.$$

Here ρ, p, u, and v are "small" quantities, calculated at the centers of the computational cells on the lower and upper time layers; R, P, U, and V are "large" quantities, determined on the boundaries of the computational cells with the help of the algorithm for the decomposition of an arbitrary discontinuity [11]; the indices 1 and 2 refer to the boundaries of neighboring cells. The temperature T and degree of ionization m are determined from the solution of the system (2) using the values of ϵ and ρ obtained when integrating the system (1).

TABLE 1

l	h	r_1	r_2	δ	ρ_0, kg/m^3	ε_0, MJ/kg
mm						
Setup 1						
75	25	2,5	22,5	10	200	185
Setup 2						
40	159	5	45	10	100	700

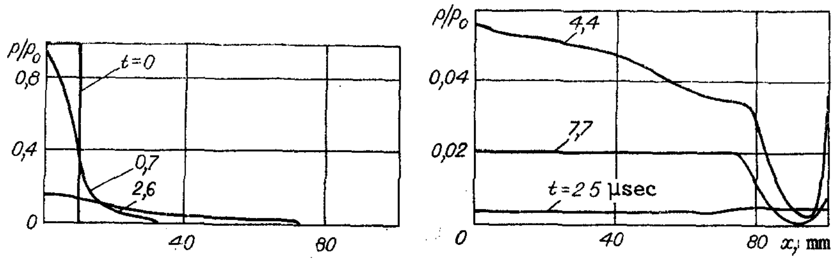

Fig. 2

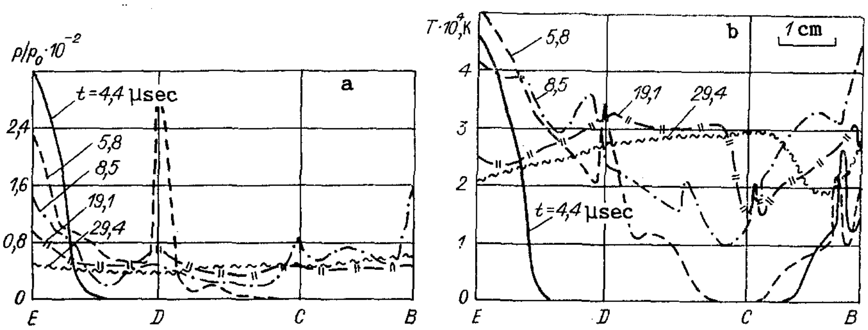

Fig. 3

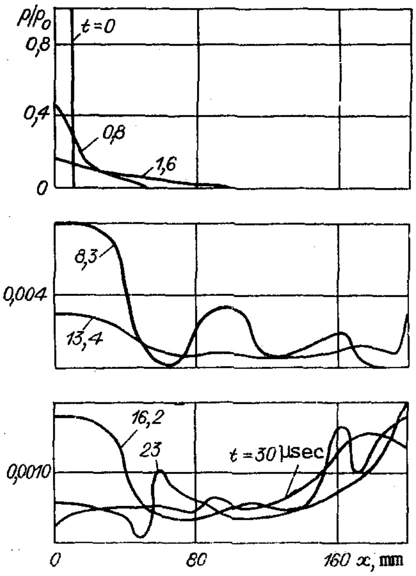

Fig. 4

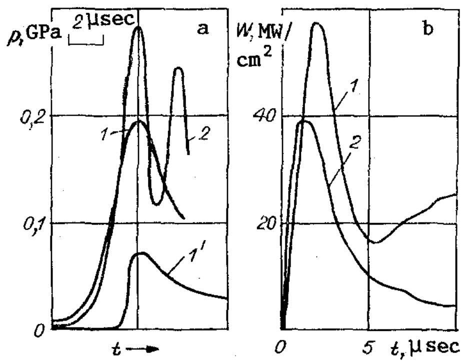

Fig. 5

The calculations are performed on a BESM-6 computer using a FORTRAN program. The maximum number of nodes in the computational grid is 100 × 45. Table 1 gives the parameters of two setups studied (see Fig. 1). The first one was employed in [2] to study intense ablation regimes and the second one was employed in [5] to study the parameters of a plasma jet. The calculations were performed for cavities filled with air under normal conditions, while for setup 2 they were also performed for a cavity evacuated to a residual pressure of 0.1 Pa. The results obtained are presented in Figs. 2-5.

Figures 2 and 4 show the distribution of the dimensionless pressure p/p_0 at different times along the symmetry axis of the setups 1 and 2 ($p_{01} = 18.5$ GPa, $p_{02} = 25.1$ GPa) with a cavity filled with air. During the starting period the flow in the pipes is strongly nonuniform and nonstationary. The duration of this period equals, in order of magnitude, $\approx \ell/c_0$ (c_0 is the starting velocity of sound in the plasmoid). Then a flow with small gradients of the pressure, density, and temperature is established in the pipe. In the latest phases, when the pressure in the pipe drops to a value less than 0.01 of the initial value, reverse flow of plasma from the cavity into the pipe is observed.

From the pipe the plasma flows with a supersonic velocity into the cavity, where the flow becomes substantially two-dimensional, and the pressure, density, and temperature drop sharply. Figure 3 illustrates the process of filling of the cavity of the setup 1 with plasma. The distributions along the perimeter of the cavity of the dimensionless pressure and temperature (a and b, respectively) are shown for different times. One-half the bottom ED, the side wall DC, and part of the cover CB are drawn along the horizontal axis. After reaching the bottom of the cavity the plasma jet forms a stagnation shock wave and starts to spread out along the bottom, and then along the walls of the cavity, forming stagnation zones with elevated pressure and temperature in the corners of the cavity. The maximum values of the parameters are obviously reached on the axis at the point of stagnation of the starting jet. The filling of the cavity with the plasma gradually approaches equilibrium. The time for establishing equilbrium filling equals, in order of magnitude, $\approx 3d/c_1$ (d is the characteristic size of the cavity and c_1 is the velocity of sound in the plasma with equilibrium filling of the cavity).

In the longer cavity of setup 2 (Fig. 4), because of radial spreading the jet strikes the lateral wall of the cavity even before it strikes the bottom; this gives rise to a local increase of the pressure.

We note that the computationally determined flow pattern gives a qualitatively correct description of the actual process. The flow in the pipe and the cavity are supersonic, the plasma flow expands as it enters the cavity, and the jet strikes the side wall of the cavity and a stagnation wave is observed at the bottom.

The computational and experimental data were compared in order to make a quantitive evaluation of the accuracy of the chosen model.

Figure 5 shows for the setup 2 the pressure at the bottom of the cavity at the point of stagnation and the hydrodynamic energy flux $W = \rho u^2/2$ at the boundary of the stagnated gas: the curves 1 show the calculation for the evacuated cavity, 1' the calculation for an air-filled cavity, and 2 the experimental data taken from [5]. In Fig. 5a time is measured on the curves from the moment of maximum pressure. The results of the calculation and experiment agree to within about 30%.

As regards the temperature in the stagnation zone of the plasma flow at the bottom of the cavity, the calculation gives a temperature that is more than two times higher than the temperature measured experimentally in [5]. This significant discrepancy is obviously explained by the fact that in the calculation the cooling of the plasma was neglected, while in the experiment the brightness temperature, which is always less than the true temperature, is measured.

The calculations performed and their comparison with experiment show that the mathematical model employed, based on Euler's equations, can be used to analyze and predict the outcome of experiments with a two-dimensional nonstationary plasma flow from an explosive generator into volumes with a complicated shape. For processes with a longer duration or for a hotter plasma, heat-transfer processes will also have to be included in the mathematical model.

LITERATURE CITED

1. A. E. Voĭtenko, "Production of high-velocity gas jets," Dokl. Akad. Nauk SSSR, 158, No. 6 (1964).

2. A. E. Voĭtenko and N. A. Tokmak, "Investigation of intense ablation regimes with help of an explosive plasma generator," Institute of Engineering Mechanics, Academy of Sciences of the Ukrainian SSR, Dnepropetrovsk (1986); VINITI, June 30, 1986, No. 4224-B26.
3. A. E. Voĭtenko, E. P. Matochkin, and A. F. Fedulov, "Explosive lamp," Prib. Tekh. Éksp., No. 2 (1970).
4. A. E. Voĭtenko and V. I. Kirko, "Efficiency of explosive plasma generators," Fiz. Goreniya Vzryva, 11, No. 6 (1975).
5. Yu. N. Kiselev, K. L. Samonin, and B. D. Khristoforov, "Parameters of an explosive gas compressor jet," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1981).
6. B. M. Growley and H. D. Glenn, "Numerical simulation of high-energy (Mach 20 to 40) air shock experiment," Proceedings of the 7th International Shock Tube Symposium, I. I. Glass, ed., Toronto University Press, Toronto (1970).
7. G. S. Romanov and V. V. Urban, "Numerical simulation of an explosive plasma generator in the gas-dynamic approximation," Inzh. Fiz. Zh., 37, No. 5 (1979).
8. G. S. Romanov and V. V. Urban, "Numerical simulation of plasma generator taking into account the energy of radiation and vaporization of the walls," Inzh. Fiz. Zh., 63, No. 6 (1982).
9. G. S. Romanov and V. V. Urban, "Calculation of the parameters of an impulsive source of light based on an explosive plasma generator," Proceedings of the 4th All-Union Conference on Dynamics of a Radiating Gas, Moscow State University, Moscow (1981), Vol. 1.
10. Ya. Yu. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow (1966).
11. S. K. Godunov (ed.), Numerical Solution of Multidimensional Problems in Gas Dynamics [in Russian], Nauka, Moscow (1976).